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Abstract: We interpret extremal non-BPS black holes in four dimensions as threshold bound states of four 1/2-BPS constituents. We verify the no-force condition for each of the primitive constituents in the probe approximation. Our computations are for a seed solution with $\overline{D0} - D4$ charges and equal B-fields, but symmetries extend the result to any U-dual frame. We make the constituent model for the $D0 - D6$ system explicit, and also discuss a duality frame where the constituents are $D3$ branes at angles. We demonstrate stability of the constituent model in the weak coupling description of the constituent D-branes. We discuss the relation between the BPS and non-BPS branches of configuration space.

Contents

1 Introduction

Non-BPS charged extremal black holes in four dimensions are interesting because they represent an intermediate step between supersymmetric extremal black holes and more physically realistic non-BPS non-extremal uncharged black holes. In this paper we explore the physical properties of non-BPS extremal black holes further with special emphasis on a constituent model for them.

In N=2 supersymmetric theories with a U-duality action on the physical fields, some properties of non-BPS extremal black holes can be obtained by analytical continuation from their BPS relatives. This is the case for the black hole entropy, perhaps the most prominent black hole characteristic. However, the black hole entropy, and analytical continuation generally, is only part of the story. Other features of known explicit black hole solutions indicate significant qualitative differences between the BPS and non-BPS branches of these U-duality invariant theories, including:

1. *Attractor behavior:* The attractor mechanism for $N = 2$ BPS black holes applies to all scalars in vector multiplets but not those in hyper multiplets. For the symmetric $N = 2$ theories which interest us there are scalars in vector multiplets that decouple from the attractor flow so that their horizon values are indeterminate. In other words, some scalars experience a flat potential. It is remarkable that it is the non-BPS black holes that exhibit the largest number of flat directions in the supergravity approximation.

2. *Mass formula and constituent model:* The mass of extremal non-BPS black hole in the supergravity limit can be written as the sum of the masses of four primitive 1/2 BPS constituents with no intrinsic entropy of their own. This property applies everywhere in moduli space, although the specific four-part split changes. The form of the mass formula suggests that, at least in the supergravity limit, all non-BPS extremal black holes are threshold bound states of four constituents. The analogous BPS mass formula is more involved so, again, there are certain remarkable cancelations that apply specifically to non-BPS black holes in the supergravity approximation.

3. *Phase diagram:* The mass of of the spherically symmetric non-BPS black holes is always strictly greater than the BPS bound, even in regions of moduli space where BPS multi-center solutions exist. This suggests that the two branches are related by a first order phase transition.

The starting point for this paper is the most general extremal static spherically symmetric non-BPS black hole solution to the STU-model. This was first constructed in [1], and later rediscovered in a different U-duality framework in [2], extending the work in [3].[1] The generating solution constructed in [2] takes a simple form in a canonical duality frame where the charges are those of anti-D0-branes and three kinds of $D4$-branes, while the three axionic scalars of the solution asymptote a common B-field. In this canonical frame the mass of the extremal black holes is:

$$2G_N M_{\text{Non-BPS}} = \frac{1}{\sqrt{2}} \left(|Q_0| + \sum_{i=1}^{3} P^i \left(1 + B^2\right) \right) , \tag{1.1}$$

with the convention that $Q_0 < 0$ on the non-BPS branch. The mass formula is simply the sum of the masses of anti-D0-branes and D4-branes individually, with the B-field taken into account for each brane independently. This suggests a constituent model with no binding energy, *i.e.* a threshold bound state. In this paper we provide further evidence in favor of this interpretation.

To appreciate how surprising the non-BPS mass formula (1.1) is, let us compare with the BPS mass formula:

$$2G_N M_{\text{BPS}} = \frac{1}{\sqrt{2}} \left| Q_0 + \sum_{i=1}^{3} P^i \left(1 + iB\right)^2 \right| , \tag{1.2}$$

---

[1]There is a lot of related work in the literature concerning first order flow equations and attractors [4] (see the review [11] and references therein), microscopics [5, 6], non-extremal D0-D6 interactions [7] and also quantum lift of flat directions [8]. The static single centered solutions studied here have also been recently extended to multi-center configurations with and without angular momentum [9, 10].

where now $Q_0 > 0$. For non-vanishing B-field the total mass is less than the sum of constituent masses. Hence we have a genuine bound state, with non-vanishing binding energy. Although the binding energy vanishes in the limit where the B-field is removed, it is believed that normalizable bound states persist in this limit which corresponds to the BPS black hole.

Our constituent model of the non-BPS black holes as a threshold bound state suggests a classical instability: throughout moduli space we should be able to remove constituent quanta from the system and take them to infinity, at no cost in energy. If we describe the effective dynamics of such a constituent quantum as a probe in the background generated by the extremal black hole, it must be the case that it feels no force, if it carries the right charges to be interpreted as a constituent of the bound state. We will test this expectation by explicit computation.

In a general U-duality frame, with arbitrary charge vector and asymptotic moduli, the non-BPS mass formula similarly takes the form of a sum of four terms, each of which is the mass of a 1/2-BPS constituent. Having identified the constituents of the non-BPS black hole in the canonical duality frame, the appropriate constituents for any other non-BPS black hole can be determined as the image under U-duality of the canonical constituents. We will make this procedure explicit for the case of the $D0 - D6$ system. In this case it is not obvious a priori what the four 1/2-BPS constituents should be. We find that the constituents of the $D0 - D6$ system are $D6$-branes with fluxes [12, 13] and verify that this gives the correct mass formula for the $D0 - D6$ in the presence of general B-fields.

We also consider a duality frame where the primitive constituents are interpreted as $D3$-branes at angles. This more geometrical setting is well-suited for discussing the spectrum of open strings stretching between the constituent branes. We will focus on the stability condition imposed by the absence of tachyons. The representation of the non-BPS black hole as $D3$-branes at angles is also well suited for discussing spacetime supersymmetry.

Extremal non-BPS black hole have an instability into just two 1/2-BPS decay products, which are not mutually local. The $D0 - D6$ frame realizes this instability in a simple manner: overall energy is lowered if the $D0$-brane tunnels and escapes to infinity. Thus our threshold bound state is at best meta-stable. The full story is in fact more interesting due to the existence of stable BPS solutions with lower energy than widely separated $D0$-brane and $D6$-branes. These configurations necessarily have multiple centers, and they exist only when a sufficiently large B-field is turned on. Whenever there is a multi-center solution available, there may be a first order phase transition between the non-BPS and the BPS branches.

This paper is organized as follows. In section 2 we review the canonical non-BPS solutions, with emphasis on the field strengths supporting the solutions. In section 3 we present the probe computation verifying that the proposed constituents feel no force from the non-BPS black hole, a delicate matter in the presence of B-fields. In section 4 we determine the constituent interpretation of the $D0 - D6$ black hole. In this setting we also discuss quantization conditions and the relation to BPS multicenter solutions. In section 5 we examine a duality frame where the primitive constituents are $D3$-branes at angles. In this setting we discuss stability of the system from the world-sheet point of view, and

we detail supersymmetry breaking. Finally, we end in section 6 with a discussion of some open issues, including some comments on the entropy of extremal non-BPS black holes.

2   The canonical non-BPS black hole

The setting for our study is the STU-model [14–16], i.e. $N = 2$ supergravity in four dimensions with $n_V = 3$ vector supermultiplets that couple through the prepotential:

$$F = \frac{s_{ijk}X^i\,X^j\,X^k}{6X^0} = \frac{X^1\,X^2\,X^3}{X^0}\,. \tag{2.1}$$

The notation is $s_{ijk} = |\epsilon_{ijk}|$ with $i, j, k = 1, 2, 3$. The STU-model is a closed subsector of both $N = 4$ and $N = 8$ supergravity and our results apply in those contexts as well, as detailed in [2].

The single center extremal black hole solutions in the STU-model are uniquely characterized by their charge vector $\Gamma = (P^I, Q_I)$ (with $I = 0, 1, 2, 3$) and the asymptotic value of the complex moduli $z^i = X^i/X^0 = x^i - iy^i$ (with $i = 1, 2, 3$). The STU-model has a $SL(2)^3$ duality symmetry that acts nontrivially on these parameters so we may consider a seed solution with just $(8 + 6) - 9 = 5$ parameters with the understanding that the most general charge vector and asymptotic moduli can be restored if needed, by acting with dualities [17].

Sufficient general black hole solutions are generally very complicated but there is a canonical duality frame where the solution simplifies [2]. In this frame the five parameters of the seed solution are four nonvanishing charges Q_0, P^i and the fifth parameter is chosen as the diagonal pseudoscalar $z^i = B - i$ (with the same B for $i = 1, 2, 3$). We take $Q_0 < 0$ and $P^i > 0$ which, in our conventions, means supersymmetry is broken. With these choices, the four dimensional metric of the seed solution is:

$$ds^2 = -e^{2U}\,dt^2 + e^{-2U}\left(dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right)\,, \tag{2.2}$$

with the conformal factor [2]:

$$e^{-4U} = -4H_0H^1H^2H^3 - B^2\,, \tag{2.3}$$

where the four harmonic functions are:

$$\sqrt{2}H_0 = -(1 + B^2) + \frac{\sqrt{2}Q_0}{r} \quad,\quad \sqrt{2}H^i = 1 + \frac{\sqrt{2}P^i}{r}\,. \tag{2.4}$$

The constants of integration have been adjusted so that the conformal factor $e^{-4U} \to 1$ as $r \to \infty$. The conformal factor is positive definite because $Q_0 < 0$. The scalar moduli z^i are written in terms of the harmonic functions as

$$z^i = \frac{B - i\,e^{-2U}}{s_{ijk}H^jH^k}\,. \tag{2.5}$$

The asymptotic behavior is $z^i \to B - i$ as $r \to \infty$ in accord with the duality frame we have chosen.

The STU-model can be interpreted as a subsector of type IIA string theory on $T^6 = T^2 \times T^2 \times T^2$. Then the scalars are the complexified Kähler moduli $z^i = x^i - iy^i$ of the three T^2's. The four electric charges correspond to $D0$ and $D2$'s wrapping the T^2's, while the magnetic charges correspond to $D6$ and $D4$'s wrapping the dual T^4's. Thus the canonical charge configuration that gives a simplified solution is the $\overline{D0}$-D4 system, with identical B-field turned on in each of the T^2's.

At this point we have not yet specified the gauge fields in the solutions. Since those play a central role in the probe computations presented in the next section, it is appropriate to derive them in detail. The result for the gauge fields is given in the end of this section.

Starting from the gauge field $\vec{\mathcal{A}}^I$ we introduce field strengths $\mathcal{F}^{\pm I} = d\mathcal{A}^I \pm i \star d\mathcal{A}^I$ ($I = 1, 2, 3$) that are imaginary anti-self-dual (imaginary self-dual) under Hodge duality. The symplectic dual field strengths defined as:

$$\mathcal{G}_{\pm J} = \overline{\mathcal{N}}_{JI} \mathcal{F}^{\pm I} \,, \tag{2.6}$$

are written analogously $\mathcal{G}_{\pm J} = d\mathcal{A}_J \pm i \star d\mathcal{A}_J$ in terms of the symplectic dual gauge field $\vec{\mathcal{A}}_J$. We decompose the field strength and the symplectic dual field strength into electric and magnetic components as:

$$d\vec{\mathcal{A}}^I = E^I \, dt \wedge dr + d\vec{a}^I \,, \tag{2.7}$$

$$d\vec{\mathcal{A}}_J = E_J \, dt \wedge dr + d\vec{a}_J \,. \tag{2.8}$$

For single center solutions the Bianchi identities for $\mathcal{F}^I$ and $\mathcal{G}_J$ determine the magnetic components uniquely in terms of conserved charges:

$$\vec{a}^I = -P^I \cos\theta \, d\phi \,, \quad \vec{a}_J = -Q_J \cos\theta \, d\phi \,. \tag{2.9}$$

The Bianchi identity for $\mathcal{G}_J$ is equivalent to the equations of motion for the "true" field strength $\mathcal{F}^I$. The magnetic charge of the symplectic dual field strength $\mathcal{G}_J$ is the electric charge in terms of $\mathcal{F}^I$ and therefore denoted Q_J.

At this point the gauge fields are completely specified but we must impose the symplectic duality condition (2.6) consistently in order to make them explicit. Taking orientation so that $\epsilon_{\hat{t}\hat{r}\hat{\theta}\hat{\phi}} = +1$ the metric (2.2) gives:

$$\star (dt \wedge dr) = e^{-2U} \, r^2 \, (d\theta \wedge \sin\theta \, d\phi) \,, \tag{2.10}$$

$$\star (d\theta \wedge \sin\theta \, d\phi) = -\frac{e^{2U}}{r^2} \, dt \wedge dr \,. \tag{2.11}$$

Note $\star^2 = -1$, as always on a four dimensional Lorentzian manifold. The Hodge duals of the magnetic fields (2.9) become:

$$\star d\vec{a}^I = -P^I \frac{e^{2U}}{r^2} \, dt \wedge dr \,, \quad \star d\vec{a}_J = -Q_J \frac{e^{2U}}{r^2} \, dt \wedge dr \,. \tag{2.12}$$

Decomposing the moduli matrix $\overline{\mathcal{N}}_{IJ}$ into its real and imaginary part, $\overline{\mathcal{N}}_{IJ} = \mu_{IJ} - i\nu_{IJ}$ and focusing on the $dt \wedge dr$ components of the 2-forms in (2.6) we solve for the electric

components $\{E^I, E_J\}$ introduced in (2.7)–(2.8) and find:

$$E^I = \frac{e^{2U}}{r^2} \nu^{IJ} \left[Q_J - \mu_{JK} P^K \right] , \tag{2.13}$$

$$E_J = \frac{e^{2U}}{r^2} \left[\mu_{JK} \nu^{KL} Q_L - \left(\mu_{JK} \nu^{KL} \mu_{LM} + \nu_{JM} \right) P^M \right] . \tag{2.14}$$

The matrix ν^{IJ} is the inverse of ν_{IJ}, i.e. $\nu^{IJ} \nu_{JK} = \delta^I{}_K$. The $d\theta \wedge d\phi$ components of (2.6) give no further constraints, they give equations that are satisfied automatically.

The expressions (2.13)–(2.14) are general, valid for $N = 2$ supergravity with any number of vector multiplets. However, they depend on the scalar fields through the moduli matrix defined as:

$$\mathcal{N}_{IJ} = \mu_{IJ} + i\nu_{IJ} = \overline{F}_{IJ} + 2i \frac{(\mathrm{Im}\, F_{IK})X^K \, (\mathrm{Im}\, F_{JM})X^M}{(\mathrm{Im}\, F_{RS})X^R X^S} , \tag{2.15}$$

$$F_{IJ} = \frac{\partial^2 F}{\partial X^I \partial X^J} , \tag{2.16}$$

and to make that dependence explicit we need the prepotential, i.e. (2.1) in the case of the STU-model. Writing out (2.15) in this case we find [16]:

$$\mu_{IJ} = \begin{pmatrix} 2x_1 x_2 x_3 & -x_2 x_3 & -x_1 x_3 & -x_1 x_2 \\ -x_2 x_3 & 0 & x_3 & x_2 \\ -x_1 x_3 & x_3 & 0 & x_1 \\ -x_1 x_2 & x_2 & x_1 & 0 \end{pmatrix} , \tag{2.17}$$

from the real part of the equation and:

$$\nu_{IJ} = y_1 y_2 y_3 \begin{pmatrix} -\left(1 + \frac{x_1^2}{y_1^2} + \frac{x_2^2}{y_2^2} + \frac{x_3^2}{y_3^2}\right) & \frac{x_1}{y_1^2} & \frac{x_2}{y_2^2} & \frac{x_3}{y_3^2} \\ \frac{x_1}{y_1^2} & -\frac{1}{y_1^2} & 0 & 0 \\ \frac{x_2}{y_2^2} & 0 & -\frac{1}{y_2^2} & 0 \\ \frac{x_3}{y_3^2} & 0 & 0 & -\frac{1}{y_3^2} \end{pmatrix} , \tag{2.18}$$

from the imaginary part. We also need the inverse of (2.18):

$$\nu^{IJ} = -\frac{1}{y_1 y_2 y_3} \begin{pmatrix} 1 & x_1 & x_2 & x_3 \\ x_1 & x_1^2 + y_1^2 & x_1 x_2 & x_1 x_3 \\ x_2 & x_1 x_2 & x_2^2 + y_2^2 & x_2 x_3 \\ x_3 & x_1 x_3 & x_2 x_3 & x_3^2 + y_3^2 \end{pmatrix} . \tag{2.19}$$

The position of the indices on the real moduli is usually taken lower (i.e. x_i, y_i) for typographical convenience although, strictly, these fields have only been defined with upper indices ($z^i = x^i - iy^i$).

The electric fields supporting the seed solution (2.2) is found from the general expressions (2.13) by inserting the moduli matrices ν_{IJ}, μ_{IJ} and turning on only the charges (Q_0, P^i). The result is:

$$E^0 = -\frac{e^{2U}}{r^2} \frac{1}{y_1 y_2 y_3} \left(Q_0 - \frac{1}{2} s_{ijk} x^i x^j P^k \right) , \tag{2.20}$$

$$E^i = -\frac{e^{2U}}{r^2} \frac{1}{y_1 y_2 y_3} \left(x^i Q_0 - (x_i^2 + y_i^2) \, s_{ijk} x_j P^k - \frac{1}{6} P^i s_{jkl} x^j x^k x^l \right) , \tag{2.21}$$

where there is no summation over the free index i in the expression for E^i. The electric fields give the forces on electric probes. The full electromagnetic field is given in (2.7) with the electric fields (2.22)–(2.21) and the magnetic fields (2.9). We will also need the dual electric fields (2.14):

$$E_0 = -\frac{e^{2U}}{r^2}\frac{1}{y^1y^2y^3}\left(-x^1x^2x^3\,Q_0 + \frac{1}{2}x_iP^i\,s_{ijk}(x_j^2+y_j^2)(x_k^2+y_k^2)\right), \qquad (2.22)$$

$$E_i = -\frac{e^{2U}}{r^2}\frac{s_{ijk}}{2y^1y^2y^3}\left(x_jx_k\,Q_0 - (x_j^2+y_j^2)(x_k^2+y_k^2)\,P^i - 2x_ix_j\,(x_k^2+y_k^2)\,P^j\right).\,(2.23)$$

Again, there is no summation over the free index i in the expression for E_i. The dual electric fields give the forces on magnetic probes.

3 Probing extremal non-BPS black holes

As explained in the introduction, it is reasonable to describe non-BPS black holes as threshold bound states of 1/2-BPS constituents. The canonical $\overline{D0}-D4$ solution reviewed in the previous section is thus interpreted as a collection of $\overline{D0}$-brane and $D4$-brane constituents placed on top of each other with no binding energy.

In this section we test the interpretation as follows. The lack of binding energy means constituents can be arbitrarily separated. Thus it should be possible to bring in additional constituents from infinity, without them being subject to a force. Accordingly, we expect $\overline{D0}$-branes and $D4$-branes wrapping any two torii to feel no force, whereas other 1/2 BPS probes like wrapped $D2$'s and $D6$'s should feel forces.

The potential felt by a static Dp-brane at a constant position due to a background field is given by the Lagrangian density of the Dp-brane, up to a sign:

$$V_{\mathrm{Dp}} = T_p\left[e^{-(\phi-\phi_\infty)}\sqrt{-\det(G+B)} - \sqrt{2}\eta\,A_{p+1}\right] \equiv T_p\,(V_{\mathrm{DBI}} + V_{\mathrm{WZ}})\,, \qquad (3.1)$$

where η parameterises whether we are describing a Dp or an $\overline{Dp}$ brane. We have in mind infinitesimal constituents being added and so it is justified to use the probe approximation where distortion of the background due to the probe is neglected. The DBI action should give a precise description even though the background is non-BPS, because the proposed constituents are 1/2-BPS.

The dilaton in (3.1) is the 10D dilaton, with its asymptotic value absorbed in the tension of the brane. The 4D dilaton is a component of a hypermultiplet, which has no radial dependence, so the 10D dilaton acquires its variation solely from the volume of T^6 in the condition $e^{-2(\phi_4-\phi_{4\infty})} = e^{-2(\phi-\phi_\infty)}V_6 = 1$. It is convenient to evaluate the combination:

$$e^{-(\phi-\phi_\infty)}\sqrt{-g_{\mathrm{tt}}} = \frac{1}{\sqrt{y^1y^2y^3}}e^U = \frac{2\sqrt{2}H^1H^2H^3}{(-I_4-B^2)}\,. \qquad (3.2)$$

Here we are introducing the conventional notation for the quartic duality invariant:

$$I_4 = 4H_0H^1H^2H^3 - 4H^0H_1H_2H_3 - \left(\sum_I H_I H^I\right)^2 + 4\sum_{i<j}H^iH_iH^jH_j\,, \qquad (3.3)$$

which the charge assignments of the seed solution reduces to:

$$I_4 \to 4H_0 H^1 H^2 H^3 . \tag{3.4}$$

Recall that $I_4 < 0$ for non-BPS solutions.

We have written the DBI-action (3.1) in the conventional manner but we should remember that the B-field appearing in (3.1) is the spatially varying B-field, whose components on each T^2 we hitherto denoted x. For a single T^2 the dictionary of notations is:

$$\sqrt{\det(G+B)} \to \sqrt{x^2 + y^2} = |z| . \tag{3.5}$$

The normalization of the WZ-term in (3.1) is unconventional, a consequence of the definition of gauge fields we adopted.[2] The contribution to the force (in units of the brane tension T_p) from the WZ term is simply:

$$-\frac{\partial V_{\mathrm{WZ}}}{\partial r} = \sqrt{2}\eta \frac{\partial A_{p+1}}{\partial r} = -\sqrt{2}\eta E . \tag{3.6}$$

The electric field one should use in this expression depends on the identity of the probe: it is E^0, E^i for $D0$, $D2$ branes and E_0, E_i for $D6$, $D4$ branes. The electric fields generated by the $\overline{D0}$-D4 background were given in (2.20)–(2.23).

We are now ready to compute the forces that the seed solution exerts on a variety of probes. In the following we establish that the $\overline{D0} - D4$ background exert no forces on $\overline{D0}$-branes, nor on $D4$-branes, despite the presence of a B-field. These results support our contention that these are the constituents of the bound state. As a means of emphasizing the nontrivial nature of the cancellations, we also carry out the corresponding computation for a BPS black hole and show that, in that case, the B-field obstructs the cancellation.

3.1 D0-brane probe

Consider a $D0$-brane (or a $\overline{D0}$-brane) experiencing the forces of the extremal non-BPS $\overline{D0} - D4$ black hole with equal B fields. The DBI contribution to the force is found by differentiating (3.2):

$$
\begin{aligned}
-\frac{\partial V_{\mathrm{DBI}}}{\partial r} &= -\frac{2\sqrt{2}H^1 H^2 H^3}{(-I_4 - B^2)^2}\left(\frac{\partial I_4}{\partial r} + (-I_4 - B^2)\sum_i \frac{1}{H^i}\frac{\partial H^i}{\partial r}\right) \\
&= \frac{1}{r^2}\frac{2\sqrt{2}H^1 H^2 H^3}{(-I_4 - B^2)^2}\left(I_4\frac{Q_0}{H_0} - B^2\sum_i \frac{P^i}{H^i}\right) .
\end{aligned}
\tag{3.7}
$$

Since $I_4 < 0$ the force is negative, i.e. towards smaller r, as one expects for the attractive gravitational and dilatonic forces. The DBI-contribution to the force is the same for a $D0$-brane and for a $\overline{D0}$-brane.

The WZ contribution to the force is given by inserting (2.20) in (3.6):

$$-\frac{\partial V_{\mathrm{WZ}}}{\partial r} = -\sqrt{2}\eta E^0 .$$

$$
\begin{aligned}
&= \frac{\sqrt{2}\eta}{r^2} \frac{8(H^1 H^2 H^3)^2}{(-I_4 - B^2)^2} \left(Q_0 - \frac{B^2}{4H^1 H^2 H^3} \sum_i \frac{P^i}{H^i} \right) \\
&= \frac{\eta}{r^2} \frac{2\sqrt{2}H^1 H^2 H^3}{(-I_4 - B^2)^2} \left(I_4 \frac{Q_0}{H_0} - B^2 \sum_i \frac{P^i}{H^i} \right).
\end{aligned}
\tag{3.8}
$$

In the second line we simplified using the second part of (3.2). The result for the WZ-force is positive for $\eta = -1$ and negative for $\eta = +1$, because $\overline{D0}$'s are repelled from the $\overline{D0} - D4$ black hole, while $D0$'s are attracted. In the case of $\overline{D0}$ the repulsion precisely cancels the attraction (3.7) such that there is no net force, even in the presence of a B-field. This result confirms our expectation that the extremal black hole contains $\overline{D0}$ constituents at threshold.

3.2 D4-brane probe

We consider a D4-brane wrapping tori one and two (without losing generality). From (3.1) we find the potential felt by a static configuration:

$$
\begin{aligned}
V_{\mathrm{DBI}} &= e^{-\phi} \sqrt{-g_{\mathrm{tt}}} |z_1| |z_2| \\
&= \frac{1}{\sqrt{2}H^3} \frac{-I_4}{(-I_4 - B^2)},
\end{aligned}
\tag{3.9}
$$

where we used (3.2) and wrote the moduli (2.5) in the useful form:

$$
|z_i|^2 = \frac{-I_4}{(s_{ijk}H^j H^k)^2}.
\tag{3.10}
$$

The DBI contribution to the force then becomes:

$$
-\frac{\partial V_{\mathrm{DBI}}}{\partial r} = \frac{1}{r^2} \frac{I_4}{\sqrt{2}H^3(-I_4 - B^2)^2} \left[-I_4 \frac{P^3}{H^3} + B^2 \left(\frac{Q_0}{H_0} + \frac{Q_1}{H_1} + \frac{Q_2}{H_2} \right) \right].
\tag{3.11}
$$

after some simplifications. Since $I_4 < 0$ the force is negative again, as one expects for the attractive gravitational and dilatonic forces.

The WZ contribution to the force is given by inserting (2.23) in (3.6):

$$
\begin{aligned}
-\frac{\partial V_{\mathrm{WZ}}}{\partial r} &= -\sqrt{2}\eta E_3. \\
&= \frac{\eta}{r^2} \frac{I_4}{\sqrt{2}H^3(-I_4 - B^2)^2} \left[-I_4 \frac{P^3}{H^3} + B^2 \left(\frac{Q_0}{H_0} + \frac{Q_1}{H_1} + \frac{Q_2}{H_2} \right) \right]
\end{aligned}
\tag{3.12}
$$

The force is positive for $\eta = -1$ so we interpret that sign as corresponding to a $D4$-brane, the case where the probe is repelled from the $\overline{D0} - D4$ black hole background. For $\eta = -1$ there is a perfect cancelation between DBI and WZ forces at all positions, and with general B taken into account. This supports our interpretation of the $D4$ as one of the 1/2-BPS constituents of the non-BPS black hole.

3.3 Other probes

Our formulae easily gives the forces on many other probe branes, such as $D2$, $D6$, and also various branes with fluxes turned on.

The case of $D2$, $D6$ is particularly simple. In the absence of a B-field there is just the attractive force due to the gravity-dilaton interactions encoded in the DBI action and the WZ-term vanishes identically because the charges involved in background and in probe are different. The inclusion of a B-field makes the accounting less transparent, because the B-field induces electric fields of all types and so a contribution from the WZ-term. Nevertheless, a net attractive force remains even when B is taken into account.

A more subtle case is when we consider $\overline{D0}$, $D4$ with fluxes on their world-volumes. The fluxes modify the DBI term and also the WZ term, obstructing the delicate cancellation exhibited above in the absence of fluxes. This gives rise to a net force on the probe. This shows that the correct constituents for the $\overline{D0} - D4$ black hole are the $\overline{D0}$'s and $D4$'s with no fluxes on their world-volumes.

3.4 A supersymmetric probe computation

The cancellation of forces made explicit in the preceding subsections is reminiscent of similar phenomena in simple supersymmetric systems. In order to appreciate that the non-BPS cancellations we exhibit are in fact novel, it is worth carrying out analogous computations for BPS black holes.

To do this let us consider the standard BPS black holes with $D0$, $D4$ charges and a diagonal B-field [18–20]. The metric remains of the form (2.2) but in the BPS case the conformal factor is:

$$e^{-4U_{\text{BPS}}} = I_4 \,, \tag{3.13}$$

where the quartic invariant I_4 defined in (3.3) depends on the harmonic functions:

$$H^0 = \overline{P}^0, \qquad\qquad H^i = \overline{P} + \frac{P^i}{r} \,,$$
$$H_0 = \overline{Q}_0 + \frac{Q_0}{r}, \qquad\qquad H_i = \overline{Q}. \tag{3.14}$$

We have $I_4 > 0$ for BPS configurations. The B-field is encoded in the constants:

$$\overline{P}^0 = \frac{1}{\sqrt{2}} \sin\alpha, \qquad\qquad \overline{Q}_0 = \frac{1}{\sqrt{2}} \left((1 - 3B^2)\cos\alpha + B(3 - B^2)\sin\alpha \right) ,$$
$$\overline{P} = \frac{1}{\sqrt{2}}(\cos\alpha + B\sin\alpha), \qquad \overline{Q} = \frac{1}{\sqrt{2}} \left(2B\cos\alpha - \sin\alpha(1 - B^2) \right) , \tag{3.15}$$

where the phase of the spacetime central charge is:

$$\tan\alpha = \frac{2B \sum_i P^i}{Q_0 + \sum_i P^i(1 - B^2)} \,. \tag{3.16}$$

The scalar fields in the BPS solution are:

$$z^i = \frac{(H^I H_I - 2H^i H_i) - ie^{-2U}}{s_{ijk} H^j H^k - 2H^0 H_i} \,, \tag{3.17}$$

– 10 –

with no sum over the index i.

Let us consider a $D0$-brane probe. In this case the DBI potential becomes:

$$
\begin{aligned}
V_{\text{DBI}} &= \frac{1}{\sqrt{y^1 y^2 y^3}} e^U \\
&= \frac{\sqrt{8(H^2 H^3 - H^0 H_1)(H^3 H^1 - H^0 H_2)(H^1 H^2 - H^0 H_3)}}{I_4} \\
&\quad \times \left(1 + 2\overline{P}\sum_i P^i \frac{1}{r}\right)\left[1 - \sqrt{2}\left((Q_0 + \sum_i P^i(1-B^2))\cos\alpha + 2B\sum_i P^i \sin\alpha\right)\frac{1}{r}\right] \\
&\sim 1 - \sqrt{2}\left(Q_0 \cos\alpha + B\sum_i P^i(\sin\alpha - B\cos\alpha)\right)\frac{1}{r}
\end{aligned}
\tag{3.18}
$$

We expanded for large r using:

$$
I_4 = 1 + \sqrt{2}\left((Q_0 + \sum_i P^i(1-B^2))\cos\alpha + 2B\sum_i P^i \sin\alpha\right)\frac{1}{r} + \cdots .
\tag{3.19}
$$

We then find the gravity-dilaton force:

$$
-\frac{\partial V_{\text{DBI}}}{\partial r} = -\sqrt{2}\left(Q_0 \cos\alpha + B\sum_i P^i(\sin\alpha - B\cos\alpha)\right)\frac{1}{r^2} + \cdots
\tag{3.20}
$$

The WZ-coupling is written in terms of the electric field in (3.6) and the applicable electric field is given in (2.22). This gives the force:

$$
\begin{aligned}
-\frac{\partial V_{\text{WZ}}}{\partial r} &= -\sqrt{2}\eta E^0 \\
&= \eta \frac{\sqrt{2}}{r^2} \frac{e^{2U}}{y_1 y_2 y_3}\left[Q_0 - (x_2 x_3 P^1 + x_3 x_1 P^2 + x_1 x_2 P^3)\right] \\
&= \frac{\eta\sqrt{2}}{r^2}\left(Q_0 - B^2 \sum_i P^i\right) + \mathcal{O}\left(\frac{1}{r^3}\right) .
\end{aligned}
\tag{3.21}
$$

The case of $\eta = 1$ corresponds to a $D0$-brane (rather than an $\overline{D0}$-brane). In this case the force cancels completely when there is no B-field, as one expect for a BPS system, but generally the B-field obstructs the cancellation:

$$
\begin{aligned}
-\frac{\partial V_{\text{WZ}}}{\partial r} - \frac{\partial V_{\text{DBI}}}{\partial r} &= \sqrt{2}\left[Q_0(1-\cos\alpha) - B\sum_i P^i(B(1-\cos\alpha) + \sin\alpha)\right]\frac{1}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right) \\
&= -\frac{2B^2}{r^2}\frac{(\sum_i P^i)^3}{(Q_0 + \sum_i P_i)^2} + \mathcal{O}\left(\frac{1}{r^3}\right) .
\end{aligned}
\tag{3.22}
$$

Thus there is generally a net force in the supersymmetric system. The force is negative, i.e. attractive, indicating that the $D0$'s are bound to the $D0 - D4$ system in the presence of a B-field. That is indeed what we expect from the BPS mass formula (1.2), which indicates that there is a genuine bound state, i.e. one with binding energy.

The point we emphasize in this section is that the analogous non-BPS state is very different from the BPS state: there is no force whatsoever even in the presence of a B-field.

4 U-duality and the $D0 - D6$ black holes

The non-BPS black hole considered so far is a seed solution. This means any other extremal non-BPS black hole solution can be generated by acting with U-duality. Acting with U-duality on the four primitive constituents identified for the seed solution, we can construct the primitive constituents appropriate for any extremal black hole we wish to analyze. By construction such primitive constituents will feel no forces from the black hole in the probe approximation. This matches the U-duality invariance of the probe potential computed before [21]. Accordingly we interpret a general non-BPS black hole as a marginal bound state of the corresponding four primitive constituents.

In this section we use employ U-duality to analyze the non-BPS extremal $D0 - D6$ black hole in the presence of background B-fields. We find that the constituents are $D6$-branes with specific fluxes turned on. In the regime with large B-fields there exist BPS configurations with the same charges as the black holes we consider. We use this circumstance to clarify the relation between the BPS and the non-BPS branches.

4.1 $D0 - D6$ constituents and the DBI mass

The five parameters of the seed solution ($\overline{D0}$-charge, three $D4$-charges, and a common B-field along three T^2's) can be mapped by U-duality to the five parameters of the $D0 - D6$ black hole ($D0$-charge Q_0, $D6$-charge P^0, and independent B-fields B_1, B_2, B_3 along three T^2's). The explicit map (constructed in section 5 of [2]) depends prominently on three parameters Λ_i related to the variables of the $D0 - D6$ frame by the equations

$$\Lambda_1 \Lambda_2 \Lambda_3 = \frac{P^0}{Q_0}, \tag{4.1}$$

$$\frac{1}{2}[\Lambda_1(1 + B_1^2) - \Lambda_1^{-1}] = \frac{1}{2}[\Lambda_2(1 + B_2^2) - \Lambda_2^{-1}] = \frac{1}{2}[\Lambda_3(1 + B_3^2) - \Lambda_3^{-1}]. \tag{4.2}$$

The awkward constraint (4.2) arises from the requirement that the B-field in the $\overline{D0} - D4$ seed solution is the same on the three T^2's. We included the factor of $1/2$ so that these expressions are precisely dual to the B-field of the seed solution.

The complete solution describing the $D0 - D6$ black hole in the presence of B-fields follows by substituting the explicit duality map into the seed solution. The resulting expressions are unwieldy and not very illuminating, so we will not present them here. A more instructive computation is to transform the four primitive constituents of the seed solution by the duality transformation and so identify the primitive constituents underlying the $D0 - D6$ black hole. This transformation gives the charge vectors

$$\Gamma_I = \frac{1}{4}\Big(P^0; -P^0/\Lambda_1, -P^0/\Lambda_2, -P^0/\Lambda_3; Q_0; P^0/(\Lambda_2\Lambda_3), P^0/(\Lambda_1\Lambda_3), P^0/(\Lambda_1\Lambda_2)\Big) \tag{4.3}$$

$$\Gamma_{II} = \frac{1}{4}\Big(P^0; -P^0/\Lambda_1, P^0/\Lambda_2, P^0/\Lambda_3; Q_0; P^0/(\Lambda_2\Lambda_3), -P^0/(\Lambda_1\Lambda_3), -P^0/(\Lambda_1\Lambda_2)\Big) \tag{4.4}$$

$$\Gamma_{III} = \frac{1}{4}\Big(P^0; P^0/\Lambda_1, -P^0/\Lambda_2, P^0/\Lambda_3; Q_0; -P^0/(\Lambda_2\Lambda_3), P^0/(\Lambda_1\Lambda_3), -P^0/(\Lambda_1\Lambda_2)\Big) \tag{4.5}$$

$$\Gamma_{IV} = \frac{1}{4}\Big(P^0; P^0/\Lambda_1, P^0/\Lambda_2, -P^0/\Lambda_3; Q_0; -P^0/(\Lambda_2\Lambda_3), -P^0/(\Lambda_1\Lambda_3), P^0/(\Lambda_1\Lambda_2)\Big) \tag{4.6}$$

in a notation where the 8 entries of the charge vectors are those of $D6$, three kinds of $D4$'s, $D0$, and three kinds of $D2$'s. The total charge vector

$$\Gamma = \Gamma_I + \Gamma_{II} + \Gamma_{III} + \Gamma_{IV} = (P^0; \vec{0}; Q_0; \vec{0})$$

is that of the $D0 - D6$ black hole, as it should be. The direct derivation of (4.3)–(4.6) can be carried out using formulae in [2] but we will also verify these expressions in section 5.1 of the present paper, using a simple duality chain.

To interpret the expressions (4.3)–(4.6) we compare with a microscopic model based on coincident D-branes. In the absence of external B-fields it has long be known [12, 13] that a total charge vector with just $D0$- and $D6$-brane charge can be reproduced using four $D6$'s wrapping $T^2 \times T^2 \times T^2$ with flux assignments:

$$(F_{12}, F_{34}, F_{56})^I = (f_1, f_2, f_3) , \tag{4.7}$$

$$(F_{12}, F_{34}, F_{56})^{II} = (f_1, -f_2, -f_3) , \tag{4.8}$$

$$(F_{12}, F_{34}, F_{56})^{III} = (-f_1, f_2, -f_3) , \tag{4.9}$$

$$(F_{12}, F_{34}, F_{56})^{IV} = (-f_1, -f_2, f_3) . \tag{4.10}$$

The superindex $\{I, II, III, IV\}$ enumerates the four $D6$'s while the subindex in the individual fluxes f_i refers to the torus in which they are thread. The induced $D0$-brane charge from the flux F is the third Chern class[3]

$$n_0 = -\frac{n_6}{4} \frac{1}{6(2\pi)^3} \int \mathrm{tr} F \wedge F \wedge F = -\frac{n_6 V_6 f_1 f_2 f_3}{(2\pi)^3} , \tag{4.11}$$

so we have

$$\frac{P^0}{Q_0} = \frac{M_6}{M_0} \frac{n_6}{n_0} = -\frac{V_6}{(2\pi)^6 \alpha'^3} \frac{(2\pi)^3}{V_6 f_1 f_2 f_3} = -\frac{1}{(2\pi\alpha')^3 f_1 f_2 f_3} . \tag{4.12}$$

In order to induce the correct total $D0$-brane charge the fluxes must be chosen so that

$$(2\pi\alpha')^3 f_1 f_2 f_3 = -\frac{Q_0}{P^0} . \tag{4.13}$$

The pattern of signs in the fluxes (4.7)–(4.10) were chosen such that all induced $D2$-brane and $D4$-brane charges cancel. The four $D6$-branes with fluxes arranged in the manner indicated are mutually local and they have the total quantum numbers expected of the primitive constituents underlying the $D0-D6$ black hole. However, the model is incomplete because it only specifies the product of the fluxes f_1, f_2, f_3, not their individual values.

The constituent charge vectors (4.3)–(4.6) determined in our construction are precisely those of $D6$-branes with fluxes (4.7)–(4.10) if we identify the parameters Λ_i with fluxes according to

$$\Lambda_i = -\frac{1}{2\pi\alpha' f_i} . \tag{4.14}$$

---

[3]The B-field contributions to the WZW term should not be included here. Our definition of charges from the four dimensional fields (2.9) gives nice quantization rules and transformation properties under U-duality but it does not include the contribution from B-fields.

As a consistency check we note that the constraint (4.1) maps to (4.13) under the identifications. More importantly, the constraints (4.1)–(4.2) determine the Λ_i's completely in terms of the charges Q_0, P^0 and the B-fields, B_i. The identifications (4.14) therefore specify the fluxes on the constituent $D6$-branes completely.

We now have the ingredients to discuss the mass of the $D0 - D6$ black hole in a background with three independent B-fields. The Dirac/Born-Infeld (DBI) mass of the $D6$ branes with flux is

$$M = T_6 \int \text{Tr} \sqrt{\det [G + (2\pi\alpha' F - B)]} \, . \tag{4.15}$$

We are not using this action in a truly non-abelian setting: we have in mind a diagonal configuration describing four branes, each of which is BPS by itself although they are not mutually BPS (they preserve different supersymmetries). Since F is a rank four bundle, we need to consider $n_6/4$ such objects to obtain the correct $D6$-brane charge. This gives a block diagonal configuration with unit metric, three B-fields, and the fluxes arranged as above. The mass becomes

$$
\begin{aligned}
M = {} & T_6 V_6 \left[(1 + (2\pi\alpha' f_1 - B_1)^2)(1 + (2\pi\alpha' f_2 - B_2)^2)(1 + (2\pi\alpha' f_3 - B_3)^2)\right]^{1/2} \\
& + T_6 V_6 \left[(1 + (2\pi\alpha' f_1 - B_1)^2)(1 + (2\pi\alpha' f_2 + B_2)^2)(1 + (2\pi\alpha' f_3 + B_3)^2)\right]^{1/2} \\
& + T_6 V_6 \left[(1 + (2\pi\alpha' f_1 + B_1)^2)(1 + (2\pi\alpha' f_2 - B_2)^2)(1 + (2\pi\alpha' f_3 + B_3)^2)\right]^{1/2} \\
& + T_6 V_6 \left[(1 + (2\pi\alpha' f_1 + B_1)^2)(1 + (2\pi\alpha' f_2 + B_2)^2)(1 + (2\pi\alpha' f_3 - B_3)^2)\right]^{1/2} . \tag{4.16}
\end{aligned}
$$

The total mass depends on the B-fields both *explicitly* as they appear in eq. (4.16) and *implicitly*, as they determine the proper choice of f_i's. The general $D0 - D6$ mass formula (4.16) computed from the DBI formula agrees with the one found (in (5.56) of [2]) by U-duality from our seed solution.

4.2 Quantization of charges and fluxes

So far we have treated fluxes and charges as continuous variables. This is reasonable for most purposes, since the objects we study are large. However, the quantization conditions lead to several important refinements which we turn to next.

One aspect of quantization is due to our constituent model having exactly four primitive constituents. For the $D0 - D6$ black hole these four constituents appear on an equal footing in that they can be permuted by symmetries. This structure is consistent with the underlying charge quantization if the underlying numbers of $D0$- and $D6$-branes both are divisible by four, but otherwise not. Non-BPS states with charges that are not divisible by four are therefore protected against spontaneous separation into primitive constituents. Generally there will be a binding energy that is finite, although not parametrically large. In other words, the binding energy will be microscopic even for macroscopic states, leaving the state fragile rather than unbound . This binding mechanism is a non-BPS version of the customary restriction to mutually prime quantum numbers for threshold BPS bound states.

Another aspect of the quantization conditions concerns the flat directions in moduli space. Recall that for non-BPS black holes there are two moduli that fail to be stabilized by the attractor mechanism, even though they are in vector multiplets. In the $D0 - D6$

frame these unfixed moduli are just ratios of the three torus volumes, v_1, v_2, v_3. The attractor mechanism does apply to the overall volume torus V_6 and the three B-field densities, so these should be kept fixed as we move along the flat directions. It is immediately apparent that the dimensionful constituent charge vectors (4.3)–(4.6) are *independent* of the flat directions.

Now, the relation between the dimensionful constituent charge vectors and the corresponding quantized charges depends on moduli. For example the dimensionless vector corresponding to (4.3) becomes:

$$\vec{n}_I = \frac{1}{4}\Big(n_6; -n_6/\lambda_1, -n_6/\lambda_2, -n_6/\lambda_3; n_0; n_6/(\lambda_2\lambda_3), n_6/(\lambda_1\lambda_3), n_6/(\lambda_1\lambda_2)\Big), \quad (4.17)$$

where $\lambda_i = \Lambda_i/v_i$. The flux densities Λ_i^{-1} remain invariant under volume changes but the scaled densities λ_i^{-1} vary. The charge split for a non-BPS bound state with the same charges as n_6 $D6$-branes and n_0 $D0$-branes therefore varies as we move along the flat direction. Moreover, the number of $D4$-branes and $D2$-branes of the primitive constituents depend continuously on the flat moduli. The lack of proper quantization implies some some interesting finite g_s corrections to our picture of the non-BPS extremal black hole as a marginal bound state, but we will not develop this point further in this paper.

4.3 Multi-center BPS solutions and decay of the non-BPS $D0 - D6$ black holes

Our extremal single-center non-BPS black hole solutions have the same quantum numbers as a certain class of multi-center BPS solutions. Some of the characteristic features of these BPS multi-center solutions are:

- Their mass is BPS, which is always *strictly* smaller than that of the non-BPS solution with otherwise identical quantum numbers (see [2]).

- They are bound states of as few as two 1/2-BPS constituents. (We interpret non-BPS solutions in terms of exactly four 1/2-BPS constituents.)

- The charge vectors of the 1/2-BPS constituents are mutually non-local, i.e. they have non-zero intersection number. (The four constituents of the non-BPS black holes are mutually local.)

- The constituents have a finite separation scale that is essentially determined by the charge intersection numbers. (The constituents of the non-BPS black holes can move freely in the supergravity approximation.)

- These BPS states only exist in part of the moduli space. There is a co-dimension one wall of threshold stability in moduli space beyond which they disappear from the spectrum. (The non-BPS black holes exist everywhere in moduli space.)

- The mutual non-locality of the charges generally necessitates angular momentum in the multi-center BPS solutions. Varying the location of the constituents gives a range of allowed angular momenta identical to the so-called "slowly" spinning non-BPS black hole.

The multi-center BPS solutions are thus very different from the non-BPS solutions analyzed in this article. For example, it is evident that the BPS solutions cannot be continuously connected to any non-BPS stationary solution through the wall of marginal stability, as illustrated in the figure below. Instead, there can be decay from the non-BPS branch to the BPS branch on the part of moduli space where BPS solutions exist. The transition will release energy, entropy and generally also angular momentum. This indicates a first order transition between the two branches.

The $D0 - D6$ duality frame is the simplest setting for making this discussion more explicit. Then the two types of constituents on the BPS branch are just $D0$-branes and $D6$-branes.[4] We will briefly summarize some of the history of BPS $D0 - D6$ bound states.

The problem of adhering $D0$-branes to $D6$-branes in a supersymmetric manner was first considered by Witten [22].[5] He found that a supersymmetric branch exists for sufficiently large B-fields

$$\sum_{i<j} B^i B^j \geq 1 \ . \tag{4.18}$$

The equality defines a wall of threshold stability, which in the equal B field case is represented as the dashed line in the figure below.

In our notation we write the condition as $\sin \alpha < 0$ where α is the phase of the spacetime central charge matrix

$$e^{i\alpha} \doteq \frac{Q_0 + iP^0 \prod_{i=1}^3 (1 + iB_i)}{\left| Q_0 + iP^0 \prod_{i=1}^3 (1 + iB_i) \right|} \ . \tag{4.19}$$

(We take $P^0, Q_0 > 0$ without loss of generality.) Near the boundary $\alpha = 0$, open strings stretching between the $D0$-branes and the $D6$-branes have light modes described by an effective quantum mechanics. The supersymmetric branch $\alpha < 0$ is the Higgs phase of this theory. Here there is a tachyonic open string mode which condenses to a supersymmetric ground state, interpreted as the $D0 - D6$ bound state. This BPS state disappears from the spectrum upon crossing the wall to $\alpha > 0$.

In the early work no BPS supergravity solution was detailed. Later progress in the field uncovered explicit supergravity BPS multi-center solutions [19, 21] which carry $D0 - D6$ charge. These are representations of the perturbative bound state in the supergravity regime. We are particularly interested in the two-center solution based on two 1/2-BPS charge vectors,

$$\begin{aligned} \Gamma_1 &= (P^0, 0, 0, 0) \,, \\ \Gamma_2 &= (0, 0, 0, Q_0) \,, \end{aligned} \tag{4.20}$$

which are not mutually local,

$$\langle \Gamma_1, \Gamma_2 \rangle = P^0 Q_0 \,, \tag{4.21}$$

but they sum up to the total $D0 - D6$ charge vector Γ. Following [21] one can show that a two-center BPS solution[6] with these charge assignments exists exactly when the separation

---

[4]The BPS branch may have other components corresponding to solutions with more than two types of constituents or with the same number of different constituents. We do not explore this possibility here.

[5]See also related work in [23].

[6]For recent work on supersymmetric D0-D6 supergravity configurations, see [24, 25].
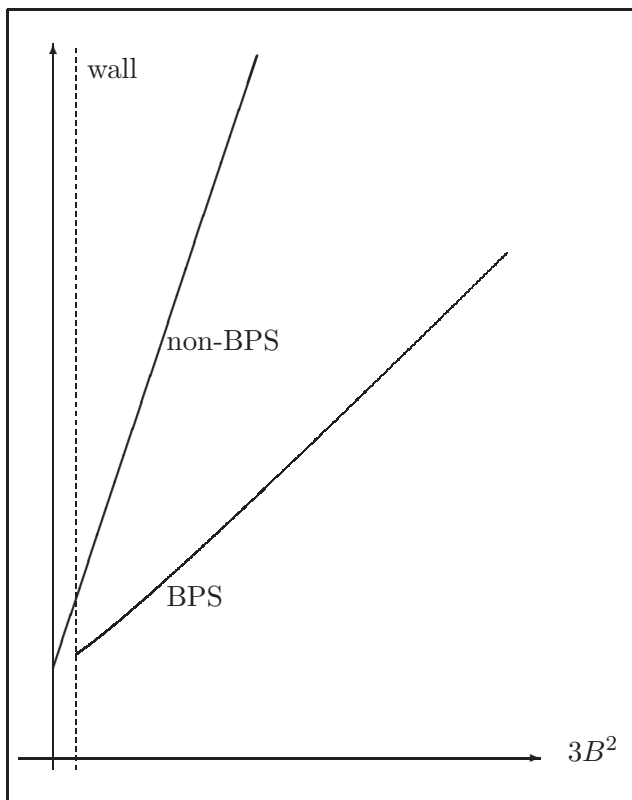
Figure 1. Mass of the non-BPS and BPS branches for diagonal B field

between the two centers is:

$$R = |\vec{x}_1 - \vec{x}_2| = -\frac{P^0}{\sin\alpha} = \frac{|Q_0 + iP^0 \prod_{i=1}^{3}(1 + iB^i)|}{\sum_{i<j} B^i B^j - 1} \ . \tag{4.22}$$

The supergravity scale (4.22) is meaningful only for $\sin\alpha < 0$, exactly the same as the condition for supersymmetry in the analysis of light open string modes.

The agreement of the BPS moduli space for the supergravity solution and the perturbative analysis can be understood as follows [21]. There are *three* regimes in parameter space where the $D0 - D6$ bound state can be simply analyzed: a "Higgs" branch, a "Coulomb" branch, and the multi-center supergravity solution. At any point on BPS moduli space, the scale (4.22) is proportional to the charges Q_0, P^0, which in terms are proportional to the coupling g_s (multiplied by the number of branes). For vanishing string coupling the scale is negligible so we can focus on the lightest open string modes which, as mentioned earlier, are tachyonic. Condensation of these modes fixes the separation scale at exactly zero. This is the Higgs branch. Any non-vanishing scale (4.22) renders the open string modes massive. This occurs whenever the string coupling is non-vanishing. As the scale becomes larger, the Higgs potential becomes more shallow, and the semiclassical approximation for the Higgs branch breaks down because the wave function spreads out more. It is the integration of these massive modes out that leads to an effective potential for the $D0 - D6$ separation

and the scale (4.22) is a consistent minimum of that potential. This is the Coulomb branch description. Finally, if the separation scale fixed by the minimum of the potential lies beyond the string scale, a supergravity analysis becomes more reliable than the quantum mechanics derived from looking at just the low-energy open string modes. In summary, increasing the string coupling g_s from zero pushes us through a cascade of useful regimes: "Higgs" to "Coulomb" to supergravity. Since this is true anywhere on the BPS moduli space $\sin \alpha < 0$ it follows that this space comes out the same at weak and strong coupling.

We can also use the scale (4.22) as a guide towards the physics near the boundary of supersymmetric moduli space. For any finite value of g_s the scale increases without bound as $\alpha \to 0^-$. Even if the fixed value of g_s is so small that the Higgs description applies initially, the motion towards the boundary $\alpha = 0$ will again force us through the cascade of useful descriptions, from "Higgs" to "Coulomb" to supergravity. For example, the Higgs branch analysis becomes unreliable as the variance in the expectation value for the tachyonic string modes becomes large. Once we reach a multi-center configuration, the separation between the $D0$-branes and the $D6$-branes diverges as we take $\alpha \to 0^-$. This justifies our contention that we are dealing with a threshold transition where BPS states completely exit from the spectrum.

The significance of this discussion for the non-BPS supergravity solutions we focus on in this paper is primarily that it falsifies an alternative hypothesis: the wall of marginal stability does *not* indicate a continuous transition to a non-BPS branch. Such a transition is also excluded on the grounds that the non-BPS branch exists for all moduli. Moreover, for the moduli $\sin \alpha < 0$ where both branches exist, the non-BPS states have larger energy, so there we expect a first order transition between the branches.

Angular momentum provides an important elaboration to this picture. The simple two-center $D0 - D6$ BPS solution discussed above is supported by angular momentum $|\vec{J}| = \frac{1}{2} P^0 Q_0$. A more general family of solutions with the same total charge vector Γ has the $D6$ at the center and places the $D0$'s on a sphere of the radius R given in (4.22) [21]. The total angular momentum of this "halo" solution is weigted by the $D6$ distribution such that it covers the entire range $0 \leq |\vec{J}| \leq \frac{1}{2} P^0 Q_0$. This is the same range of allowed angular momenta found for the "slowly rotating" extremal non-BPS black hole [26]. The first order transition from the non-BPS to the BPS branch can therefore proceed for any of these allowed angular momenta, and for the entire range of moduli with $\sin \alpha < 0$.

There are other similarities between the two branches. For example, it is interesting that the multi-center BPS solutions share the same flat directions as their extremal non-BPS black hole cousins. At first this may seem to contradict our previous statement about BPS black holes being complete attractors. However, the multi-center solutions do not have regular horizons, they have primitive 1/2-BPS components with singular behavior at their "horizons". This structure is precisely what is needed to allow an orbit of multi-center solutions under the same non-compact group as for the extremal non-BPS black hole.

5 Non-BPS black holes from $D3$-branes at angles

In order to be sufficiently general, our generating $\overline{D}0 - D4$ solution must allow for an ambient B-field, and the dual $D0 - D6$ black holes must have 3 independent B-fields. There is yet another useful duality frame, where all the charges of the black holes are those of $D3$-branes. In this frame the inevitable additional parameter is incorporated geometrically, as a relative angle of the intersecting $D3$-branes.

As an application of the $D3$-brane representation we discuss the perturbative open string analysis of the intersecting $D3$-brane system. We also discuss the supersymmetry preserved by various subsets of $D3$-branes, but broken by the system as a whole.

5.1 Duality from $D0 - D6$ to $D3$-branes at angles

As we have emphasized, it is advantageous to analyze the $D0 - D6$ system in terms of its four primitive constituents with charge vectors (4.3)–(4.6). These are four constituent $D6$-branes with fluxes, which we turn into four constituent $D3$-branes by acting with T-duality three times, once along each of the three T^2's. On each T^2, T-duality along one direction turns the flux into the angle that the resulting brane subtends with respect to the transverse direction.

To be more precise, the world-volume theory of a D-brane is determined entirely by the gauge invariant quantities

$$2\pi\alpha'\mathcal{F}_i = 2\pi\alpha' f_i - B_i = -(\Lambda_i^{-1} + B_i)\,, \tag{5.1}$$

where we used the notation (4.14) for the inverse fluxes. The resulting angle with respect to the undualized direction on the ith T^2 is then determined by

$$\cot\phi_i = 2\pi\alpha'\mathcal{F}_i = -(\Lambda_i^{-1} + B_i)\,. \tag{5.2}$$

In our set-up the T-duality acts on each of the four constituent branes enumerated by $A, B = I, II, III, IV$ and endowed with the fluxes (4.7)–(4.10). For each pair of branes the fluxes are the same on one of the three T^2's and it has the opposite sign on the remaining T^2's. If the fluxes agree, the branes are obviously parallel within that T^2, i.e. their relative angle is $\vartheta_i^{AB} = \phi_i^A - \phi_i^B = 0$. If the fluxes have opposite signs, the relative angle of the brane pair within that torus is

$$\cot\vartheta_i^{AB} = \cot(\phi_i^A - \phi_i^B) = \frac{1 - \cot(\phi_i^A)\cot(\phi_i^B)}{\cot(\phi_i^A) + \cot(\phi_i^B)} = -\frac{1}{2}\left[\Lambda_i(1 + B_i^2) - \Lambda_i^{-1}\right]\,. \tag{5.3}$$

Here Λ_i refers to brane A, in contrast to $\Lambda_i^B = -\Lambda_i$. At this point we recall that the parameters Λ_i are determined by charges and B-fields according to (4.1)–(4.2). In fact the constraints (4.2) demand that the expression on the right hand side of (5.3) is the same for each of the T^2's.

$$\cot\vartheta_i^{AB} = -\frac{1}{2}\left[\Lambda_i(1 + B_i^2) - \Lambda_i^{-1}\right] \equiv -b\,. \tag{5.4}$$

In summary, after duality each pair of constituent $D3$-branes have the same relative angle (5.4) within two of the T^2's, and they are parallel within the last T^2.

In the preceding formulae we have been sloppy with signs, in order to keep notation simple: in (5.1) we took the field strength f_i on the ith T^2. According to the flux assignments (4.7)–(4.10) this is accurate for the constituent $A = I$ but for $A = II, III, IV$ some of the fluxes actually have their signs flipped. The overall sign in our final formula for the relative angles (5.4) should be adjusted accordingly.

The constant b has a simple interpretation, mentioned already after (4.2). It is the b-field in the $\overline{D0} - D4$ seed solution that gave rise to the $D0 - D6$ black hole after dualities. It now appears in the expressions for the angles subtended by $D3$-branes. We can take the dualities full cicle by aligning our coordinate system with one of the $D3$-branes, and then dualize it to a $D0$ brane. Under this duality the remaining $D3$-branes turn into $D4$-branes. This sequence of dualities can be viewed as an independent derivation of the $D0 - D6$ primitive constituents (4.3)–(4.6). In particular, it gives confidence in the nonlinear constraints (4.2) imposed on the fluxes in the $D0 - D6$-frame.

5.2 Perturbative stability of extremal black holes

We can study the stability of the extremal non-BPS black holes at weak string coupling by examining the open strings stretching between its four constituent D-branes. We will do this in the $D3$-brane duality frame where the fluxes are encoded in the angles between the branes. The corresponding discussion for the $D0 - D6$ system (and other frames) then follow by duality.[7]

We first consider a single pair of $D3$-branes on T^6. Each of the $D3$'s have one direction on each of the three T^2, and their relative angles on the three T^2's are θ_i with $0 \leq \theta_i \leq \pi$. The boundary conditions on the open strings stretching between the $D3$'s are then twisted due to the relative angles. For example, the complex scalars on the three T^2 have fractional modes $X^i_{n+\theta_i/\pi}$ and fermion modes are similarly twisted. Summing up the twisted ground states energies and keeping only the GSO projected states, the lightest states in the NS-sector become four complex scalars in spacetime with masses (for more details see e.g. [27]):

$$
\begin{aligned}
\alpha' m_1^2 &= \frac{1}{2\pi} \left(-\theta_1 + \theta_2 + \theta_3 \right) , \\
\alpha' m_2^2 &= \frac{1}{2\pi} \left(\theta_1 - \theta_2 + \theta_3 \right) , \\
\alpha' m_3^2 &= \frac{1}{2\pi} \left(\theta_1 + \theta_2 - \theta_3 \right) , \\
\alpha' m_4^2 &= 1 - \frac{1}{2\pi} \left(\theta_1 + \theta_2 + \theta_3 \right) .
\end{aligned}
\tag{5.5}
$$

It is clear that for some values of the relative angles the perturbative spectrum contains tachyons, interpreted as a classical instability. A pair of constituent $D3$-branes is stable if

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[7]The duality is from $D3$-branes at angles to the *constituents* of the $D0 - D6$-system. We can also apply formulae similar to the ones below directly to the $D0 - D6$ branes (following [22]) but in that case all three angles are turned on, giving rise to tachyons and spacetime supersymmetry breaking.

their relative angles satisfy the *stability conditions*:

$$\theta_2 + \theta_3 \geq \theta_1 \,,$$
$$\theta_1 + \theta_3 \geq \theta_2 \,,$$
$$\theta_1 + \theta_2 \geq \theta_3 \,,$$
$$2\pi \geq \theta_1 + \theta_2 + \theta_3 \,, \tag{5.6}$$

which assure the absence of tachyons. The non-BPS extremal black hole is perturbatively stable if these conditions are satisfied for any pair of constituents.

The stability conditions are generally quite complicated. However, as we summarized after (5.4), the relevant angles are very simple in our case: for each pair of branes one of the relative angles vanish, while the other two angles have identical norm which we can choose in the range $0 \leq |\vartheta_i^{AB}| \leq \pi$. Identifying $\theta_i = |\vartheta_i^{AB}|$, the stability conditions (5.6) are easily verified. In more detail, the spectrum of light open strings (5.5) takes the values

$$\alpha' m^2 = 0 \,, 0 \,, \frac{\theta}{\pi} \,, 1 - \frac{\theta}{\pi} \,, \tag{5.7}$$

for each pair of branes, where θ is the only non-trivial angle with $0 \leq \theta \leq \pi$. These masses are non-negative.

In the R-sector the ground state energy vanishes by world-sheet supersymmetry. The four complex fermion oscillators $\psi_{n+\theta_i/\pi}^i$ (with $i = 0, 1, 2, 3$) then have positive frequency modes $0, 0, \theta/\pi, 1 - \theta/\pi$. The GSO projected states have exactly one fermion oscillator so we recover the spectrum (5.7), in accordance with spacetime supersymmetry satisfied by each pair of $D3$-branes.

In the $D3$-frame we have seen that the equality of the relative angles on two different tori gives a simple solution to the stability conditions (5.6) for each pair. The stability conditions are less transparent in other duality frames. In the $D0 - D6$ frame, the dictionary (5.3) identifies the relative angles with the obscure combinations $-\frac{1}{2}[\Lambda_i(1 + B_i^2) - \Lambda_i^{-1}]$. The requirement (4.2) that these combinations be the same on the three T^2's can therefore be interpreted as a stability condition in the $D0 - D6$ frame.

Considering the second equation in (5.4) we can alternatively identify the relative angles of the $D3$-branes on the three T^2's with the background B-fields in the $\overline{D0} - D4$ seed solution. The absence of open string tachyons among pairs of the $\overline{D0} - D4$ branes is thus upheld by the choice of a diagonal B-field in this frame.

The perturbative open string analysis evidently focusses on pairs of constituent D-branes. However, as we detail in the following section, the total breaking of supersymmetry can be seen only when all four 1/2-BPS constituents are taken into account. Since each of the pairs preserves some supersymmetry, the absence of tachyons from the open string spectrum was therefore anticipated on general grounds. The complete spectrum of the non-BPS black hole includes collective states that depend for their existence on the presence of three and four branes. Only the last kind, depending on all four constituent branes, are sensitive to supersymmetry breaking. Since the classical non-BPS black hole entropy vanishes unless all four constituent branes are present, we expect numerous modes of this

type. The finite entropy and sensible thermodynamics do not suggest any instability among these more exotic modes so we expect any tachyons in this sector either.

The instability to the BPS branch discussed in section 4.3 is not manifested as a tachyon even among the modes depending on the presence of four branes, because this instability is nonlocal in character: the non-BPS states can be represented geometrically as four $D3$-branes intersecting over complex lines, while the corresponding BPS states are just two $D3$-branes intersecting over a single point. In the region of moduli space where the BPS bound state exists, both 3-surfaces have locally minimal areas, so transition from one type of surface to the other will be non-perturbative. This is consistent with the first order phase transition we discussed in section 4.3.

5.3 Supersymmetry breaking of $D3$-branes at angles

The black holes we study are non-BPS but their primitive constituents are 1/2-BPS. It is interesting to examine how supersymmetry is broken by the addition of the different components of the bound state. Our discussion will be in the $D3$ duality frame for definiteness but the supersymmetry structure is the same for the four primitive constituents in other frames.

In order to compare the preserved supercharges of one $D3$-brane with those preserved by another, we must rotate the supercharges. The generators of rotations within the T^2's are:

$$S_1 = \frac{i}{2}\,\Gamma^1\Gamma^2\,, \qquad S_2 = \frac{i}{2}\,\Gamma^3\Gamma^4\,, \qquad S_3 = \frac{i}{2}\,\Gamma^5\Gamma^6\,. \tag{5.8}$$

It is useful to organize the preserved supercharges into eigenvectors of the rotation generators [28, 29]:

$$S_i|s_1\,,s_2\,,s_3\rangle \,=\, \frac{1}{2}s_i\,|s_1\,,s_2\,,s_3\rangle\,,$$

where $s_i = \pm$. Each supercharge $|s_1\,,s_2\,,s_3\rangle$ is also a chiral spinor in the two non-compact dimensions, and so corresponds to two supercharges. Thus a single $D3$-brane preserves 16 supercharges, 1/2 of the maximal supersymmetry.

Consider a pair of constituent $D3$-branes of type A, B situated at the relative angles $\vartheta_1^{AB}, \vartheta_2^{AB}, \vartheta_3^{AB}$ within the three T^2's. A candidate supersymmetry $|s_1\,,s_2\,,s_3\rangle$ preserved by one of these is preserved also by the other when

$$s_1\vartheta_1^{AB} + s_2\vartheta_2^{AB} + s_3\vartheta_3^{AB} \,=\, 0 \bmod 2\pi\,. \tag{5.9}$$

When applying these conditions we must be careful with the *sign* of the angle ϑ_i^{AB}. We will follow the definition (5.4) when determining relative signs.

Consider for example the constituents I and II. The fluxes (4.7)–(4.8) mean the corresponding $D3$-branes are aligned on the first T^2 so $\vartheta_1^{I,II} = 0$. On the remaining T^2's the $D3$-branes generally meet at a nontrivial angle. In fact they meet at the *same* relative angle $\vartheta_2^{I,II} = \vartheta_3^{I,II}$ on the remaining T^2's. The supersymmetry condition (5.9) then correlates the spinorial indices of the supersymmetries on those last T^2's: both $D3$-branes preserve $|s_1\,,+\,,-\rangle$ and $|s_1\,-\,+\rangle$ for $s_1 = \pm$. Proceeding similarly for the others pairs of the form (I, B) we find the results reported in table 1.

Pair	Alignment angle	Supersymmetry
(I, II)	$\vartheta_1^{I,II} = 0$	$\lvert s_1, -, + \rangle, \lvert s_1, +, - \rangle$
(I, III)	$\vartheta_2^{I,III} = 0$	$\lvert -, s_2, + \rangle, \lvert +, s_2, - \rangle$
(I, IV)	$\vartheta_3^{I,IV} = 0$	$\lvert -, +, s_3 \rangle, \lvert +, -, s_3 \rangle$

Table 1. (I, B) pairs of constituents and their preserved supersymmetries.

Pair	Alignment angle	Supersymmetry
(III, IV)	$\vartheta_1^{III,IV} = 0$	$\lvert s_1, -, - \rangle, \lvert s_1, +, + \rangle$
(II, IV)	$\vartheta_2^{II,IV} = 0$	$\lvert -, s_2, - \rangle, \lvert +, s_2, + \rangle$
(II, III)	$\vartheta_3^{II,III} = 0$	$\lvert -, -, s_2 \rangle, \lvert +, +, s_3 \rangle$

Table 2. (B, C) pairs of constituents and their preserved supersymmetries.

The remaining pairs of $D3$-branes have the opposite relative orientation. For example, consider the constituents III and IV. The fluxes (4.9)–(4.10) again correspond to $D3$-branes that are aligned in first T^2 so $\vartheta_1^{III,IV} = 0$. However, they meet at the *opposite* relative angles $\vartheta_2^{III,IV} = -\vartheta_3^{III,IV}$ on the remaining T^2's. The supersymmetry condition (5.9) again imposes no condition on the first T^2 but now it *anti*-correlates the remaining T^2's, leaving the preserved supersymmetries $\lvert s_1, -, - \rangle$ and $\lvert s_1, +, + \rangle$ for $s_1 = \pm$. Proceeding similarly for the remaining pairs we find the results reported in table 2.

Each pair of $D3$-branes preserve 8 supersymmetries, 1/4 of maximal supersymmetry. We see from the tables that any three constituent $D3$-branes still preserve four supersymmetries, 1/8 of maximal supersymmetry. For example the triple (I, II, III) preserves $\lvert -, -, + \rangle$ and $\lvert +, +, - \rangle$. It is when we add the fourth constituent that supersymmetry will be broken. For example, the supersymmetries preserved by the pair (I, II) have none in common with those preserved by the pair (III, IV).

Multiple branes intersecting at angles generally preserve supersymmetry exactly when all the branes are related by SU(3) rotations [28–30]. In the examples above, the rotation relating each pair of branes is indeed SU(2) with respect to an appropriate complex structure. Also, the rotations relating each triple fit into an SU(3) for some complex structure. The important point is that the required complex structures are incompatible. For example, the rotations with $\vartheta_2^{III,IV} = -\vartheta_3^{III,IV}$ are U(1) rotations with opposite angles, and so they combine to an SU(2) rotation with respect to the most obvious complex structure. The rotations with $\vartheta_2^{I,II} = \vartheta_3^{I,II}$ similarly combine to an SU(2) rotation, but with respect to a complex structure that has the opposite orientation on one or the other T^2.

The preceding analysis was for generic rotation angles ϑ_i^{AB}. Alternatively we can try

to satisfy the supersymmetry conditions (5.9) by considering special angles. If for each pair we can take one of the angles ϑ_i^{AB} to vanish and the other two identical, either 0 or π, then *all* the 16 supersymmetries of a single $D3$-brane are preserved. The dictionary (5.4) shows that these special angles appear in the limit $b = \pm\infty$. Therefore full supersymmetry is restored in our $\overline{D0} - D4 - D4 - D4$ seed solution with a diagonal B-field B, in the limit where $B \to \pm\infty$.

6 Discussion

We conclude with a few open questions that we hope to address in future work:

- **Quantum corrections:** Our constituent model has no binding energy at the classical level, and it has exactly flat directions that are special to the non-BPS branch. It is interesting to ask whether these properties are preserved by quantum corrections.

  One indication that they are not is the apparent failure (discussed in section 4.2) of charge quantization: the black hole itself carries quantized charges, but it is generally not clear why the charges of the individual constituents should be. This is related to the breaking of the classical U-duality group $(\mathrm{SL}(2, R))^3$ to its discrete version by quantum corrections. This relation may give a way to understand the corrections more precisely.

- **The 5D interpretation:** in the absence of B-fields the $D0 - D6$ solution allows a a purely geometrical interpretation in $5D$, identified as a near horizon patch of the extremal Kerr solution [5]. It would be interesting to extend this identification to include the three B-fields. This will introduce charges in 5D and for this larger family of solutions there may be limits that are under good control.

- **Multi-center solutions:** our arguments suggest the existence of multi-centered extremal non-BPS supergravity configurations satisfying two basic requirements: the locations of the centers should not be constrained and the charge vector at each center should should be that of the appropriate constituent model.

  The multi-center non-BPS solutions reported in [31] seem to confirm this expectation, for the special case of diagonal charges. It would be interesting to check this expectation for generic moduli in the more general configurations described in [9, 10] and to study the differences that will arise in the presence of angular momentum.

- **The microscopic interpretation of non-BPS black hole entropy:** classically, the entropy formulae of BPS and non-BPS black holes are almost identical. For example, the black holes in $N = 8$ supergravity have entropy of the form $S_{\mathrm{BPS}} = 2\pi\sqrt{|J_4|}$ with the non-BPS/BPS distinction encoded in the sign of the quartic invariant J_4. The obvious similarity between the entropy formulae on the two branches is commonly interpreted as a hint that their microscopic origins are virtually identical, i.e. related by analytical continuation [5]. Our work challenges this interpretation in its simplest form, by highlighting significant differences between the two branches. For example,

the classical moduli spaces are different even in their dimensionality. This is relevant because precise counting of BPS states often involves choosing a favorable point in moduli space, such as turning on a small B-field to avoid bound states at threshold. This is not possible for the non-BPS states where the B-field is a classical modulus. Therefore the corresponding microstates cannot be related by analytical continuation.

Despite these challenges we remain sympathetic to the idea that the extremal non-BPS entropy can be understood in a simple manner. The significant differences between the two branches must be addressed by a more detailed understanding of the microscopics. Indeed, they may give guidance towards such a description.

Acknowledgments

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